Some Basic Misconceptions in Algebra

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Why a theory of learning?

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A useful learning theory should:

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*** predict what misconceptions learners might develop**

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*** explain how & why these misconceptions develop**

*** assist learners in resolving misconceptions**

Teachers are often skeptical of theory - they want something practical ...

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"... in the end, there is nothing as practical as a good theory." - Dewey

Role of Theory (Davis, 1984)

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In 1640's, terrible drought in Italy. Water table had dropped, and their pumps could no longer suck/pull the water to the surface. So they tried better & better pumps, but still FAILURE!

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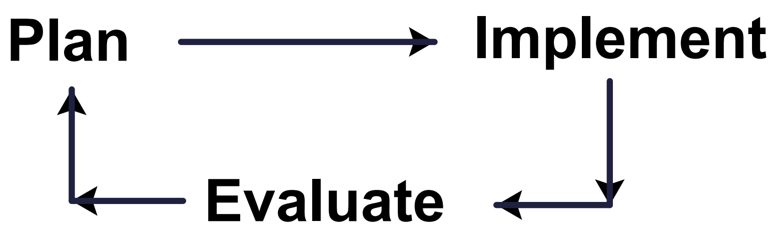
In 1643, Torricelli presented an alternative explanation/theory. The pumps did not suck up the water, but they created a vacuum in the pipes & it was atmospheric pressure that pushed the water up the pipes. But this won't work in a well deeper than 10 m! Led to invention of hydraulic pumps to solve the problem.



Not only learner performance evaluated, but also desired aims & objectives as well as effectiveness of teaching methods

Focus Question

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How effective are current teaching startegies with regard to the development of concepts like variable and function?

Different Meanings of Letter Symbols within Mathematics

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* as specific unknown numbers in equations, e.g $2x + 3 = 13$

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* as representative of arbitrary numbers or generalized number to describe general properties of numbers, e.g as in an identity $a(b + c) = ab + ac$

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* as numerical variables in algebraic expressions or functions; e.g. $2x + 3$ or $y = 2x + 3$

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How do learners interpret & handle letter symbols in algebra?

Letter symbol as variable

* prerequisite for conceptually understanding functions

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* understanding that there is a functional relation between input (x) and output (y)

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* understanding how $3x + 2$; $3x + 3$, $2x + 3$, x^2, $x^2 + 1$ 'behave' differently

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* forms basis of interpreting/drawing graphs, increasing/decreasing, extreme values, domain and range, rate of change, max/min, etc.

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* essential for modeling of real world contexts with algebraic functions

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RUMEUS Test Grade 8-9 results (1984)

Q3. Which is the larger, $2n$ or $n + 2$?
a) $2n$ b) $n + 2$ c) they are equal d) one cannot say

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Large numbers of students chose options a, b or c.

similar results from students-professor (Q22), taxi (Q42-46), days-weeks problems Q47-49, etc.

→ shows learners have difficulty working with letter symbols as generalised number or variable

Letter symbol as generalized number

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Q21. When is the following true? L + M + N = L + P + N
a) always b) never c) sometimes, when ...

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72% of learners chose b) never (because $M \neq N$)
(different letters stand for different numbers)

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→ shows these view letter symbols as specific unknowns (with *different letters* for *different numbers*)

Letter symbol as specific unknown

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Q6. Add 4 added to 3n ...

No of students 'ignored' the letter, writing 7 as the 'answer'

Q9. If $e + f = 8$, then $e + f + g = ...$?

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* No of students assigned specific values to the letter symbols, writing 15 since *g* is the 7th letter in the aplhabet
* operation is closed/conjoined: $8 + g = 8g$

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Letter symbol as object

Many learners had this view, e.g. seeing '*a*' as an abbreviation for 'apple' or as letters of alphabet: 3 *a*'s and 4 a's make 7 a's

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Many teachers seem to encourage this approach to prevent learners from 'closure' and 'conjoining' algebraic expression, e.g. for something like $2a + 5a + 3b$ using the analogy of 'apples' and 'bananas' so that learners get $7a + 3b$ instead of $10ab$.

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But how effective is this teaching strategy?

Not really effective in preventing closure: Closure occurred at more or less 20 questions, ranging from 3% to 16% in some!

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But the worst effect of 'letter as object' view was in modeling type questions involving interpreting the meaning of algebraic expressions and functions, e.g Q22, Q47-48

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Q14. Apples cost 8 cents each and pears 6 cents each. If a stands for the number of apples bought and p stands for the number of pears bought, what does 8a + 6p stand for?
Q15. Complete: The total number of apples & pears bought =

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Majority of students thought

*** 8*a* meant '8 apples' instead of cost of *a* apples at 8c each!**

*** 8*a* meant 'cost of 8 apples' instead of ...**

Q55. Choose your own letters to write an algebraic equation for the following:
Two pencils and magazine cost 90c.
Q56. What do the letters you used stand for?

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Majority of learners chose p and m and wrote the correct equation: $2p + m = 90$
BUT they interpreted p and m as 'pencils' and 'magazines respectively, or as 'number of pencils' and 'number of magazines'
instead of as 'cost/price of 1 pencil' and 'cost/price of 1 magazine' respectively!

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Getting correct answers, but show no understanding!

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For the majority of learners in the RUMEUS study it seemed that they did not interpret letter symbols as numbers, but had a 'fruit salad algebra' view.

Possible research agendas on application of Van Hiele theory to Algebra & Functions

FOR GEOMETRY

* **explanatory** and **descriptive** of **encountered problems**
* **prescriptive** regarding **ordering** of geometry

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Four important characteristics - Usiskin (1982:4):

- **fixed order** - The order is **invariant**. Learner cannot be at level *n* without having passed through level *n*-1.
- **adjacency** - Each level **builds on** preceding level
- **distinction** - Each level has **own linguistic symbols** and own **network of relationships** connecting those symbols.
- **separation** - Two persons who reason at **different levels** cannot understand each other.

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1) Which of these characteristics apply in the context of algebra and functions?
2) How can these characteristics be empirically investigated?

3. Do any these 'levels', Input/output, Flow diagram, Table, Ordered pairs, Verbal description, Algebraic equation, Graph, form a hierarchy?

4. Pre-test-post-test research design? E.g. evaluate learners understanding before and after? Can any improvement, or not, be specifically ascribed to the VH levels?